UNIT – IV

8. (a) Prove that inverse and transpose of an orthogonal matrix are orthogonal.

   (b) With the help of matrices solve the equations $x + 2y + 3z = 4$, $x + 4y + 9z = 6$, $x + y + z = 3$

9. Diagonalize the matrix A and hence find A^8, where

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Roll No. ....

3007

B. Tech. 1st Semester (ME) Examination – February, 2022

MATH - I (Calculus and Matrices)

Paper : BSC-MATH-101-G

Time : Three Hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Unit. Question No. 1 is *compulsory*. All questions carry equal marks.

1. (a) Prove $\beta(m, n) = \beta(n, m)$.

   (b) Evaluate the integral $\int_0^1 \int_0^{x^2} 7e \, dy \, dx$.

   (c) Test the convergence of the series :

$$1 + \frac{1}{4^{2/3}} + \frac{1}{9^{2/3}} + \frac{1}{16^{2/3}} + \text{.........}$$

   (d) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then compute curl $\vec{r}$ = ?

(e) Let $f : R^2 \to R$ defind as $f(x, y) = x^2 + y^2$, then compute $\lim\limits_{(x \to y) \to 0} f(x, y) = ?$

(f) Using Cayley-Hamilton theorem find A^n if :

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

UNIT – I

2. (a) Examine for maximum and minimum value of the function $f(x, y) = 3x^2 - y^2 + x^3$.

(b) Show that :

$$\log(x+h) = \log x + \frac{h}{x} - \frac{h^2}{2x^2} + \ldots\ldots + (-1)^{n-1}\frac{h^n}{n(x+\theta h)^n}.$$

3. (a) Find the volume of solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(b) Prove that $\gamma(n + \frac{1}{2}) = \frac{\sqrt{\pi}\gamma(2n+1)}{2^{2n}\gamma(n+1)}$.

UNIT – II

4. (a) Show that the series $\sum\limits_{n=1}^{\infty}\frac{1}{n^p}$ converges if $p > 1$ and diverges if $0 < p \leq 1$.

(b) Discuss the convergence of the series :

$$1 + \frac{x}{2} + \frac{\lfloor 2}{3^2}x^2 + \frac{\lfloor 3}{4^3}x^3 + \frac{\lfloor 4}{5^4}x^4 + \ldots\ldots \infty ; x > 0$$

5. (a) Expand $\log(1+z)$ about $z = 0$ as a Taylor's series.

(b) Develop $\sin\left(\frac{\pi x}{l}\right)$ in half-range cosine series in the range $0 < x < l$.

UNIT – III

6. (a) Show that the function :

$$f(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{(x^2 + y^2)}, & (x, y) \neq (0, 0) \\ 0, & \text{otherwise} \end{cases}$$

is continuous at (0, 0).

(b) If $u = \tan^{-1}\left(\frac{x}{y}\right)$, verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

7. (a) Examine maxima and minima for the function :

$$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$$

(b) Find the directional derivative of $\text{div}(\vec{v})$ at the point (1, 2, 2) in the direction $2x\hat{i} + 2y\hat{j} + 2z\hat{k}$, where $\vec{v} = x^4\hat{i} + y^4\hat{j} + z^4\hat{k}$.

P. T. O.